

Quantum erasure in the presence of a thermal bath: the effects of system-environment microscopic correlations

A. R. Bosco de Magalhães* and J. G. Peixoto de Faria†

Departamento de Física e Matemática

Centro Federal de Educação Tecnológica de Minas Gerais

Av. Amazonas 7675, Nova Gameleira

Belo Horizonte, MG, CEP 30510, Brazil

R. Rossi Jr.‡

Universidade Federal de Viçosa — Campus Florestal

Florestal, MG, CEP 35690-000, Brazil

Abstract

We investigate the role of the environment in a quantum erasure setup in the cavity quantum electrodynamics domain. Two slightly different schemes are analyzed. We show that the effects of the environment vary when a scheme is exchanged for another. This can be used to estimate the macroscopic parameters related to the system-environment microscopic correlations.

PACS numbers: 03.65.Ta, 42.50.Pq, 03.65.Yz, 03.75.Dg

Keywords: quantum erasure, cross decay rates, open quantum systems, cavity quantum electrodynamics

*Electronic address: magalhaes@des.cefetmg.br

†Electronic address: jgpfaria@des.cefetmg.br

‡Electronic address: romeu.rossi@ufv.br

I. INTRODUCTION

“In reality, it contains the only mystery, the basic peculiarities of all of quantum mechanics.” [1]. The famous statement by Richard Feynman about the wave-particle duality, gives a glance on the relevance of the subject to quantum theory. The double slit experiment illustrates very well the wave-particle duality. In such an experiment, if the information about which slit the quanton (in the sense of [2, 3]) has crossed (which-way information) is available, the interference fringes are not visible on the screen (particle behavior); however, if the which-way information is not available, there is an interference pattern (wave behavior).

At the Solvay conference (1927), A. Einstein presented a *gedanken* experiment (the Recoiling Slit Experiment) which consisted in a double slit experiment with a movable slit placed before the double slit. The goal was to detect which-way information of the quanton (recorded by the movable slit) and still see an interference pattern [4]. The apparent difficulty imposed by such *gedanken* experiment was solved by N. Bohr, who pointed out that a careful analysis of the movable slit would require the inclusion of uncertainty relations of its position and momentum; this would add random phases in the quanton path and consequently it would make the interference pattern vanish. Therefore, in this argumentation, N. Bohr used the uncertainty principle to sustain the wave particle duality.

In the eighties, another *gedanken* experiment, the quantum eraser, proposed by M. Scully and collaborators [5–7], brought back to light the debate about wave-particle duality. In the quantum eraser experiments, the quanton interacts with a probe system, and they become entangled. This interaction makes the which-way information available and destroys the interference pattern, even when there is no relevant modifications on the quanton position and momentum degrees of freedom. According to the authors, the entanglement is the essential key behind this phenomenon, and it is not necessary to call upon Heisenberg’s uncertainty principle, as it was done in the early discussions between A. Einstein and N. Bohr. As a result, a debate on the role of the entanglement and uncertainty relations began [8–12]. In a quantum eraser experiment, the which-way information available in the entangled state can be erased, and consequently the interference pattern recovered, by correlating the measurement results of the probe and the interferometric system.

Several experimental observations of the quantum eraser have been reported [13–18]. The quantum eraser is an important tool for debating on fundamental questions, but it is also

used in practical applications. To quote a few examples: In Ref. [19], it was used as a tool for channel corrections; in Ref. [20], to improve the cavity spin squeezing; in Ref. [21], for imaging applications; in Ref. [22] for experimental entanglement verification.

In the present work, we propose an experimental setup in the context of cavity quantum electrodynamics (CQED) where quantum erasure can be accomplished. We propose an implementation of Ramsey interferometry, where the interferometric paths are represented by two internal states of Rydberg atoms. The which-way information is held by a bipartite system composed of two microwave modes that interact with a common environment modeled by a thermal reservoir [23, 24].

The correlation resulting from the interaction between two or more systems and a common bath is responsible for the appearance of a set of states that are robust against decoherence [25, 26]—the decoherence-free subspaces. In our model, the coupling of the bipartite system to a common bath leads to a dramatic difference in the process of erasure depending on the class of states chosen to perform it. This fact allows us to propose a measurement scheme of the parameter related to the cross correlations resulting from the interaction between the bipartite which-way register and the environment.

This work is organized as follows. In the next section, we describe the basic setup and compute the action of the bath. In Section 3, we describe how a slightly modified scheme can be used to highlight cross decay rates related to the cavity modes. The Conclusion is found in Section 4.

II. SETUP FOR QUANTUM ERASURE

In the Ramsey interferometry experiments, the interference is observed by dealing with the states of the internal degrees of freedom of atoms or molecules: the role of different paths in this type of interferometry is played by them. Accordingly, it is necessary to create coherent superpositions of these states with the ability to manipulate the relative phase. When counting the number of atoms or molecules in a given state as a function of the relative phase, the interference can be observed. Clearly, the which-way information destroys the interference, and the erasure of this information may be used to restore it. The proposed experiment is based on Ramsey interferometry. We examine a setup where there is which-way information related to the states of two microwave modes. The scheme involving

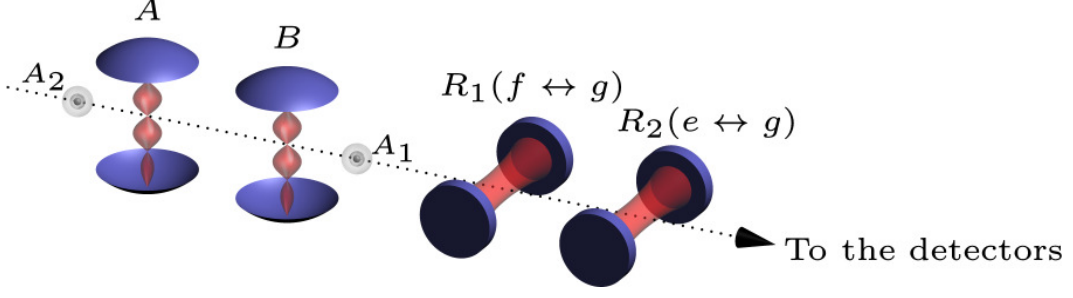
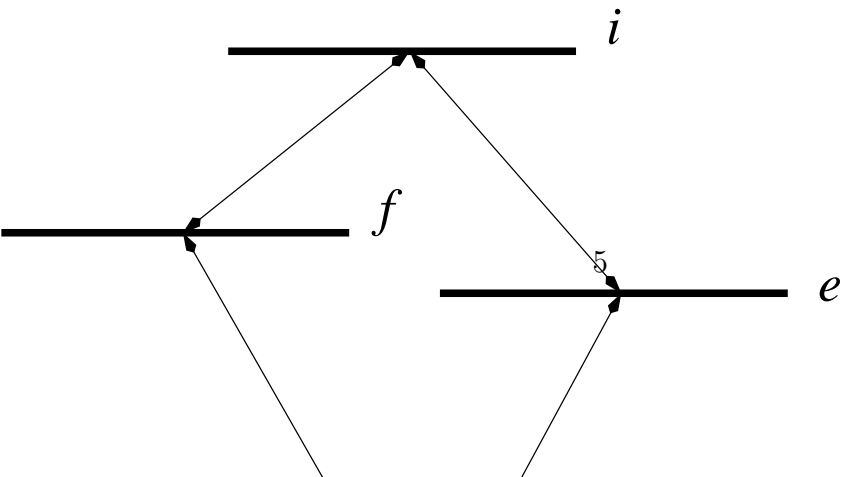


Figure 1: CQED experimental setup devised to detect the quantum erasure assisted by the environment. A Ramsey interferometer is implemented using two microwave cavities, R_1 and R_2 , where classical fields are stored, resonant or quasi-resonant with transitions $f \leftrightarrow g$ and $e \leftrightarrow g$, respectively. A and B are high-Q microwave cavities that work out as path identifiers. The atomic levels e and g play the role of interferometric paths of the atom A_1 . The second atom A_2 is used to probe the state of the high-Q cavities A and B , and the erasing of the which-path information is yielded by coincidence measurements of the states of the two atoms.

two modes is more complex than that possibly designed with only one mode; nevertheless, as will be clear, two cavity modes are necessary to investigate system-environment microscopic correlations.

Consider two superconducting cavities A and B that support the resonant modes M_A and M_B with frequency ω . Atoms with levels i , e , f , and g relevant to the experiment will go through these cavities and two Ramsey zones (see Fig. 1). The frequencies related to the atomic transitions are illustrated in Fig. 2. The tuning of the atomic transitions with the field modes can be performed by means of the Stark effect. The transition $e \rightarrow g$ is assumed to be resonant with the modes M_A and M_B when there is no Stark effect. We assume, in what follows, that the relations between couplings and detunings are such that non resonant transitions can be ignored at every step.

We consider the initial state of the cavities as the vacuum state. An atom prepared in the state i is sent, first passing through the cavity A and then the cavity B . When the atom enters the cavity A , the $i \rightarrow e$ transition is brought into resonance with the mode M_A by the Stark effect during one $\pi/2$ Rabi pulse. Next, it goes to the cavity B , where the transition $i \rightarrow f$ is put into resonance with the mode M_B for one π Rabi pulse. Then, the atom flies to a Ramsey zone tuned with the transition $f \rightarrow g$. After this transition is performed, the atom travels to another Ramsey zone, tuned with the transition $e \rightarrow g$. With a suitable



choice of the atomic dipole, the state of the system just before this Ramsey zone will be

$$|\psi_1\rangle = \frac{|g_1\rangle |0_A\rangle |1_B\rangle + e^{i\phi_1} |e_1\rangle |1_A\rangle |0_B\rangle}{\sqrt{2}}, \quad (1)$$

where ϕ_1 depends on the energies of the modes and atomic states, as well as the distances in the experimental apparatus and the velocity of the atom. When the atom passes through the second Ramsey zone, the system evolves to

$$|\psi_2\rangle = \frac{1}{2} \left[|e_1\rangle (|0_A\rangle |1_B\rangle + e^{i\phi_1} |1_A\rangle |0_B\rangle) - |g_1\rangle (|0_A\rangle |1_B\rangle - e^{i\phi_1} |1_A\rangle |0_B\rangle) \right]. \quad (2)$$

The probability of finding the atom in the state e (g) is given by $P_e = 1/2$ ($P_g = 1/2$), showing no interference.

In ordinary Ramsey interferometers, the aim in the first step is to create a state of superposition between the atomic levels e and g with a relative phase that can be varied. The preparation of the state $|\psi_1\rangle$ is analogous to this first step. However, this state clearly exhibits a perfect path discrimination due to the entanglement between the atom and the cavity modes. This prevents the direct observation of the interference between the paths related to e and g as in usual Ramsey interferometers. To observe the interference, it is necessary to perform the erasure of the which-way information. In the original proposal for quantum erasure, this was performed by a detector that interacted only with the symmetric mode of the field. Here, the erasure is achieved by sending a second atom that absorbs only the energy of the symmetric mode or of the antisymmetric mode of the field. As we will see, the action of the environment can vary according to this choice.

In order to investigate how the bath can disturb the erasure process, we permit a time interval τ between the end of the interaction of the field with the first atom and the beginning of the interaction with the second atom. The environment, at zero temperature, will be considered only during this interval, which should be large compared to the other times involved in the experiment. The decay of the fields and atomic states should be slow enough so that they can be neglected outside this interval. Experiments with atoms with slow decay and cavities with very high quality factors have been reported [27].

For the bath at zero temperature, the action of the environment can be computed using the master equation

$$\frac{d}{dt}\rho = \mathcal{L}\rho.$$

Here, ρ refers to the state of the field modes M_A and M_B and the Liouvillian \mathcal{L} is given by

$$\begin{aligned}\mathcal{L} \cdot = & -i [\omega a^\dagger a + \omega b^\dagger b, \cdot] \\ & + k (2a \cdot a^\dagger - \cdot a^\dagger a - a^\dagger a \cdot) + k (2b \cdot b^\dagger - \cdot b^\dagger b - b^\dagger b \cdot) \\ & + k_c (2a \cdot b^\dagger + 2b \cdot a - \cdot b^\dagger a - \cdot a^\dagger b - a^\dagger b \cdot - b^\dagger a \cdot),\end{aligned}\tag{3}$$

where a^\dagger (b^\dagger) and a (b) are the creation and annihilation operators related to the modes M_A (M_B), respectively, k is the decay rate of both the cavity modes, and k_c is the cross decay rate. We use the usual notation for superoperators, where the symbol \cdot indicates where the density operator must be placed. According to the discussion presented in [28], we consider $k_c \geq 0$.

Let us return to the experimental sequence. Suppose that the state of the first atom is measured immediately after it passes through the second Ramsey zone. If the result of this measurement is e , the state of the field modes after the interval τ will be

$$\rho_{M_A, M_B, e} = (\zeta_+ |1_A\rangle |0_B\rangle + \eta_+ |0_A\rangle |1_B\rangle) (H.c.) + (1 - |\zeta_+|^2 - |\eta_+|^2) |0_A\rangle |0_B\rangle \langle 0_A| \langle 0_B|,$$

where *H.c.* stands for *Hermitian conjugate* and

$$\begin{aligned}\zeta_+ &= \frac{e^{i\phi_1} f(\tau) + l(\tau)}{\sqrt{2}}, & \eta_+ &= \frac{f(\tau) + e^{i\phi_1} l(\tau)}{\sqrt{2}}, \\ f(t) &= \frac{e^{-i\omega t}}{2} (e^{-(k+k_c)t} + e^{-(k-k_c)t}), & l(t) &= \frac{e^{-i\omega t}}{2} (e^{-(k+k_c)t} - e^{-(k-k_c)t}).\end{aligned}$$

Then, a second atom, initially in the state g , absorbs the antisymmetric field mode by interacting with the mode M_A during three π Rabi pulses and with the mode M_B for one $\pi/2$ Rabi pulse. With respect to M_A , the interaction time can be adjusted by removing the atomic transition from the resonance with this mode, by means of the Stark effect, in the beginning of the path of the atom inside the cavity A , and then by letting in the resonance for the time necessary for three π Rabi pulses. As regards M_B , one can allow the atom to interact with this mode during the time required for the $\pi/2$ Rabi pulse, in the beginning of its path inside the cavity B , after which the interaction is interrupted using the Stark effect. When the atom leaves the cavity B , M_A is in the vacuum state and the second atom plus M_B state can be written as

$$\begin{aligned}\rho_{M_B, A_2, e} &= \frac{1}{2} \{ [e^{-i\phi_2} (\zeta_+ - \eta_+) |e_2\rangle |0_B\rangle + (\zeta_+ + \eta_+) |g_2\rangle |1_B\rangle] [H.c.] \\ &\quad + 2 (1 - |\zeta_+|^2 - |\eta_+|^2) |g_2\rangle |0_B\rangle \langle g_2| \langle 0_B| \},\end{aligned}$$

where $e^{-i\phi_2}$ corresponds to the phase accumulation during the Stark effect in cavity B . If we consider that the first atom was measured in the state g , we reach a final state concerning the second atom and the mode M_B given by

$$\begin{aligned} \rho_{M_B, A_2, g} = & \frac{1}{2} \left\{ [e^{-i\phi_2} (\zeta_- - \eta_-) |e_2\rangle |0_B\rangle + (\zeta_- + \eta_-) |g_2\rangle |1_B\rangle] [H.c.] \right. \\ & \left. + 2 (1 - |\zeta_-|^2 - |\eta_-|^2) |g_2\rangle |0_B\rangle \langle g_2| \langle 0_B| \right\}, \end{aligned}$$

where

$$\zeta_- = \frac{-e^{i\phi_1} f(\tau) + l(\tau)}{\sqrt{2}}, \quad \eta_- = \frac{f(\tau) - e^{i\phi_1} l(\tau)}{\sqrt{2}}.$$

Therefore, the probabilities of measuring the first atom in the state x and the second one in the state y (where x and y stand for e or g) are

$$\begin{aligned} P_{ee} &= \frac{1}{4} (1 - \cos \phi_1) e^{-2(k-k_c)\tau}, \\ P_{eg} &= \frac{1}{2} - P_{ee}, \\ P_{ge} &= \frac{1}{4} (1 + \cos \phi_1) e^{-2(k-k_c)\tau}, \\ P_{gg} &= \frac{1}{2} - P_{ge}. \end{aligned} \tag{4}$$

In equations (4) we see that the interference is completely recovered in two cases. One of them corresponds to $\tau = 0$, i.e., there is no interaction with the environment. The other is the limiting case $k_c = k$, where the environment does not disturb the erasure process, since it interacts only with the symmetric mode [28] and the erasure is based on the absorption of the antisymmetric mode.

III. INVESTIGATING THE CROSS DECAY RATES

In the sequence described above, the second atom absorbs energy only from the antisymmetric mode. If we change this scheme so that the second atom interacts with the mode M_A during the time of one π Rabi pulse, and maintain the remainder unmodified, it will absorb energy from the symmetric mode only. In this case, the limiting case $k_c = k$ leads to the maximum attenuation of the interference fringes. Indeed, the probabilities of measuring the

state of both atoms, defined analogously as in equation (4), are

$$\begin{aligned}
P'_{ee} &= \frac{1}{4} (1 + \cos \phi_1) e^{-2(k+k_c)\tau}, \\
P'_{eg} &= \frac{1}{2} - P'_{ee}, \\
P'_{ge} &= \frac{1}{4} (1 - \cos \phi_1) e^{-2(k+k_c)\tau}, \\
P'_{gg} &= \frac{1}{2} - P'_{ge}.
\end{aligned} \tag{5}$$

The interference decreases according to $e^{-2(k-k_c)\tau}$ in equations (4), and according to $e^{-2(k+k_c)\tau}$ in equations (5); this result is related to the fact that the environment acts more strongly on the symmetric mode than on the antisymmetric mode [28]. This can be used to measure the cross-decay rate k_c . Once the two experimental schemes have been completed, the measures of the frequency of the atomic states can be aggregated by computing the quantity

$$\begin{aligned}
\xi &= P_{ge} - P_{gg} - P_{ee} + P_{eg} + P'_{ge} - P'_{gg} - P'_{ee} + P'_{eg} \\
&= e^{-2(k-k_c)\tau} (1 - e^{-4k_c\tau}) \cos \phi_1,
\end{aligned}$$

which will be non-zero if, and only if, k_c is not null.

IV. CONCLUSION

We explored the effects of the environment on quantum erasure in the cavity quantum electrodynamics domain. We showed that the bath disturbs the erasure process in a way that may depend on the details of the experimental setup. In fact, the attenuation of the interference fringes due to the environment depends, for non zero cross decay rates, on the mode (symmetric or antisymmetric) absorbed by the eraser, namely, the second atom that crosses the apparatus. This can be used to estimate the cross decay rates, which are related to microscopic correlations in the system-environment interaction. These rates, whose conditions for existence may be associated with the construction of modes spatially close in the scale of their wavelengths, are responsible for superradiance and subradiance and, in the limit, for decoherence-free subspaces. In this limit, the antisymmetric mode decouples from the bath, and an erasure scheme not affected by the environment can be envisaged.

V. ACKNOWLEDGEMENTS

ARBM and JGPF acknowledge the support from the Brazilian agencies CNPq (Grants 486920/2012-7, 305380/2012-5 and 306871/2012-2) and CEFET/MG (PROPESQ program—Grant 10122_2012). RRJr acknowledges the Brazilian agency FAPEMIG (Grant APQ-00597-12) for partial financial support.

-
- [1] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* McGraw-Hill, New York, 1965.
 - [2] B.G. Englert, *Phys. Rev. Lett.* **77**, 2154 (1996).
 - [3] J.-M. Lévy-Leblond, *Physica B* **151**, 314 (1988).
 - [4] N. Bohr, Albert Einstein: Philosopher-Scientist, *Library of Living Philosophers*, Evanston, pp. 200–241 (1949).
 - [5] M. O. Scully and K. Drühl, *Phys. Rev. A* **25**, 2208 (1982).
 - [6] M. O. Scully and H. Walther, *Phys. Rev. A* **39**, 5229 (1989).
 - [7] M. O. Scully, B. G. Englert, and H. Walther, *Nature* **351**, 111 (1991).
 - [8] E. P. Storey, S. M. Tan, M. J. Collett, and D. F. Walls, *Nature* **367**, 626 (1994).
 - [9] H. M. Wiseman and F. E. Harrison, *Nature* **377**, 584 (1995).
 - [10] R. Mir, J. S. Lundeen, M. W. Mitchell, A. M. Steinberg, J. L. Garretson, and H. M. Wiseman, *New J. Phys.* **9**, 287 (2007).
 - [11] A. Luis and L. L. Sánchez-Soto, *Phys. Rev. Lett.* **81**, 4031 (1998).
 - [12] Paul Busch and Christopher Shilladay, *Physics Reports* **435**, 1–31 (2006).
 - [13] S. Dürr, T. Nonn, and G. Rempe, *Nature* **395**, 33 (1998).
 - [14] G. Teklemariam, E. M. Fortunato, M. A. Pravia, T. F. Havel, and D. G. Cory, *Phys. Rev. Lett.* **86**, 5845 (2001).
 - [15] G. Teklemariam, E. M. Fortunato, M. A. Pravia, Y. Sharf, T. F. Havel, D. G. Cory, A. Bhattaharyya, and J. Hou, *Phys. Rev. A* **66**, 012309 (2002).
 - [16] T. J. Herzog, P. G. Kwiat, H. Weinfurter and A. Zeilinger, *Phys. Rev. Lett.* **75**, 3034 (1995).
 - [17] S. P. Walborn, M. O. Terra Cunha, S. Pádua, and C. H. Monken, *Phys. Rev. A* **65**, 033818 (2002).

- [18] A. Gogo, W. D. Snyder, and M. Beck, Phys. Rev. A **71**, 052103 (2005).
- [19] Francesco Buscemi, Phys. Rev. Lett. **99**, 180501 (2007).
- [20] Ian D. Leroux, Monika H. Schleier-Smith, Hao Zhang, and Vladan Vuletic, Phys. Rev. A **85**, 013803 (2012).
- [21] J. Ajimo, M. Marchante, A. Krishnan, A. A. Bernussi, and L. Grave de Peralta, J. Appl. Phys. **108**, 063110 (2010).
- [22] E. Togan, Y. Chu, A. S. Trifonov, L. Jiang, J. Maze, L. Childress, M. V. G. Dutt , A. S. Sørensen, P. R. Hemmer, A. S. Zibrov, and M. D. Lukin, Nature **466**, 730 (2010).
- [23] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).
- [24] A. R. Bosco de Magalhães, Phys. Lett. A **375**, 4120 (2011).
- [25] G. Massimo Palma, Kalle-Antti Suominen, and Artur K. Ekert, Proc. R. Soc. London Ser. A **452**, 567 (1996).
- [26] Lu-Ming Duan and Gang-Can Guo, Phys. Rev. A **57**, 737 (1998).
- [27] S. Gleyzes, S. Kuhr, C. Guerlin, J. Bernu, S. Deléglise, U. B. Hoff, M. Brune, J. M. Raimond, and S. Haroche, Nature **446**, 297 (2007).
- [28] R. Rossi Jr., A. R. Bosco de Magalhães, and M. C. Nemes, Phys. Lett. A **356**, 277 (2006).